K S SCHOOL OF ENGINEERING & MANAGEMENT

DEPARTMENT OF APPLIED SCIENCE

Discrete Mathematics Structures / BCS405A

QUESTION BANK

MODULE I
FUNDAMENTALS OF LOGIC

1. **Explain** the following with an example each
   (a) Proposition (b) Tautology (c) Contradiction (d) Contingency (e) logical connectives (f) Rule of universal specification.
2. **Examine** whether the compound proposition is logical equivalent using truth tables:
   a. $[p \rightarrow (q \wedge r)] \Leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$
   b. $[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$
3. **Prove** that the following compound proposition is tautology or not using truth table.
   a). $[(p \vee q) \wedge \{(p \rightarrow r) \wedge (q \rightarrow r)\}] \rightarrow r$
   b). $\{p \rightarrow (q \rightarrow r)\} \rightarrow \{(p \rightarrow q) \rightarrow (p \rightarrow r)\}$
4. **Prove** the following by using the laws of logic:
   a. $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$
   b. $[\neg p \wedge (\neg q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$
   c. $[(p \vee q) \wedge (p \vee \neg q) \vee q] \Leftrightarrow p \vee q$
   d. $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$
5. **Prove** the validity of the following arguments:

   a. No engineering student of first or second semester studies logic
   Anil is an engineering student who studies logic

   ∴ Anil is not in second semester

   b. If Ravi studies, then he will pass in DMS
   If Ravi does not play cricket, then he will study
   Ravi failed in DMS

   ∴ Ravi played cricket

   c. **Test** the validity of the following arguments:
   (a). $p \wedge q$
   $q \rightarrow r$
   $r \rightarrow s$

   $\therefore s$

6. Let p, q, r be the propositions having truth values 0,0, and 1 respectively. Find the truth values of the following's propositions:
   a. $(p \vee q) \vee r$
   b. $(p \wedge q) \rightarrow r$
   c. $P \wedge (r \rightarrow q)$
   d. $p \rightarrow (q \rightarrow \neg r)$

7. **Prove the following argument is valid:**

$$\forall x, [p(x) \vee q(x)]$$
$$\exists x, [\neg p(x)]$$
$$\forall x, [\neg q(x) \vee r(x)]$$
$$\forall x, [s(x) \rightarrow \neg r(x)]$$
$$\overline{\therefore \exists x [\neg s(x)]}$$

(b)

$$p \rightarrow q$$
$$q \rightarrow (r \wedge s)$$
$$\neg r \vee (\neg t \vee u)$$
$$p \wedge t$$
$$\overline{\therefore u}$$

8. **i) State** converse, inverse and contrapositive for the condition along with **necessary and** sufficient condition.

   If a quadrilateral is a parallegram then its diagonals bisect each other

   If a triangle is not isosceles, then it is not equilateral.

   (ii) Define (a) open sentence (b) quantifiers.

   For the following statements, the universe comprises of all non-zero integers. **Determine the** truth values of each statement by considering the following open statements:

   p(x): $x \geq 0$, q(x): $x^2 \geq 0$ r(x): $x^2 - 3x - 4 = 0$ s(x): $x^2 - 3 > 0$

   (i) $\exists x, p(x) \wedge q(x)$ (ii) $\forall x, p(x) \rightarrow q(x)$ (iii) $\forall x, q(x) \rightarrow s(x)$

   (iv) $\forall x, r(x) \vee s(x)$ (v) $\exists x, p(x) \wedge r(x)$ (vi) $\forall x, r(x) \rightarrow p(x)$.

9. Establish the validity of the following argument using the rules of inference:

   $$\{ p \wedge (p \rightarrow q) \wedge (r \vee s) \wedge (r \rightarrow \neg q) \rightarrow (s \vee t) \}$$

10. P.T i). direct proof ii) proof by contradiction for the following statement:

    "If n is an odd integer then n+9 is an even integer.

MODULE 2
PROPERTIES OF INTEGERS

1. **Prove** by mathematical induction

   a. $1 + 5 + 9 - - - - - - - + (4n - 3) = n(2n - 1)$.

   b. $4n < n^2 - 7$ *for all integers* $n \geq 6$.

   c. ***Prove that*** $3 + 3^2 + 3^3 + - - - - - - - +3^n = 3(3n - 1)/2$

   d. **Show that** $1^2 + 2^2 + - - - - - - - +n^2 = n(n + 1)(2n + 1)/6$

   e. **Show that** $1^2 + 3^2 + - - - - - - - +(2n - 1)^2 = n(2n - 1)(2n + 1)/3$

2. **Determine** the coefficient of $a^2 b^3 c^2 d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$.
3. **How many** arrangements are there for all letters in the word SOCIOLOGICAL? In how many of there arrangement i)A&G are adjacent ii)All the vowels are adjacent.
4. **Find** the coefficient:

   i) x^{12} in the expansion of $x^3(1 - 2x)^{10}$.

ii) xyz^2 in the expansion of $(2x-y-z)^4$

5. **Find** the number of signals that can be generated using six different colored flags when any number of them may be hoisted at any time.
6. A certain question paper contains 3parts A,B,C with 4 questions in part A, 5 questions in part B and 6 questions in part C. It is required to answer 7 questions selecting at least two questions from each part. In **how many** ways can a student select his seven questions for answering?.
7. In **how many** ways can10 identical pencils be distributed among 5children in the following cases: i)No container is left empty.

   ii)The fourth container gets an odd number of balls
8. In **how many** ways can 10 identical pencils be distributed among 5children in the following cases: **i)**There are no restrictions.

   ii)Each children gets at least one pencil

   iii)The youngest child gets atleast two pencils.

9. In **how many** ways 10 roses,14 sunflowers,15 daffodils can be distributed among 3 girls?
10. **State** the pigeonhole principle and generalization of pigeonhole principle. **Prove that** if 30 dictionaries in a library contains a total of 61,327 pages, then at least one of the dictionaries must have at least 2045 pages.
11. How many persons must be chosen in order that at least seven of them will have birthday in the same calendar month?

    i)There are no restrictions.

    ii)Each children gets at least one pencil

    iii)The youngest child gets atleast two pencils.

12. In **how many** ways 10 roses,14 sunflowers,15 daffodils can be distributed among 3 girls?

MODULE 3

RELATIONS AND FUNCTIONS

1. Let A = 1,2,3,4 and B=1,2,3,4,5,6

   (a)Find how many functions are there from A to B. How many of these are one to one? How many are onto?

   (b)Find how many functions are there from B to A. How many of these are one to one? How many are onto?

2. Let A={1,2,3} and B={2, 4, 5}.**Determine** the following.
   (a) $|AXB|$
   (b) Number of relation from A to B
   (c) Number of relation from B To A
   (d) Number of relations from A TO b that contains (1,2) and (1,5)

3. Let A={1,2,3,4,5,6,7} and B={w, x, y, z}. **Find** the number of onto function from A to B.

   (i) Let $f: R \longrightarrow R$ be defined by

$$f(x) = \begin{cases} 3x-5 & for\ x>0 \\ -3x+1 & for\ x \leq 0 \end{cases}$$

   a. **Determine** $f(0), f(-1), f(5/3), f(-5/3)$.
   b. **Find** $f^{-1}(0), f^{-1}(1), f^{-1}(-1), f^{-1}(3), f^{-1}(-3), f^{-1}(-6)$
   c. **What is** f^{-1}([-5, 5]) and f^{-11}([-6, 5])?

4. Let f, g, h be functions from Z to Z defined by $f(x) = x - 1$,

$$g(x) = 3x, h(x) = \begin{cases} 0, if\ x\ is\ even \\ 1, if\ x\ is\ odd \end{cases}$$

Determine $(f \circ (g \circ h)(x))$ and $((f \circ g) \circ h)(x)$ and verify that $f \circ (g \circ h) = (f \circ g) \circ h$

5. Consider the function $f: R \to R$ defined by $f(x) = 2x + 5$.

   Let a function $g: R \to R$ be defined by $g(x) = 12(x - 5)$. **Prove that** g is an inverse of f.

6. Let $A = \{1,2,3,4,6\}$and R be the relation on A defined by aRb iff a is a multiple of b.
   i)Write down R ii)Draw its digraph iii) Write the matrix of R.

7. For a given set A= {1,2,3,4} ,let R be a relation on A:

   R={(1,2)(1,3)(1,4)(2,3)(2,4)(3,4)(2,1)(3,1)(4,1)}.

   a. Draw digraph of R
   b. Determine the indegree and outdegree of the vertices of digraph.
8. Draw Hasse diagram representing the positive divisors of 36.
9. For a fixed integer n>1, **Prove that** the relation 'congruent modulo n' is an equivalence relation.
10. Consider the set A= {1,2,3,4,5} and the equivalence relation:
    R={(1,1)(2,2)(2,3)(3,2)(3,3)(4,4)(4,5)(5,4)(5,5)} define onA. **Find** the partition of A induced by R
11. Let A= {1,2,3,4,5}.Define a relation R on A x A by $(x_1,y_1)R(x_2,y_2)$ iff $x_1+y_1=x_2+y_2$
    i)**Verify that** R is an equivalence relation on A x A.

    ii) ***Determine the*** equivalence classes [(1,3)],[(2,4)] and [(1,1)].

MODULE 4
THE PRINCIPLE OF INCLUSION AND EXCLUSION

1. In **how many** ways the 26 letters of English alphabet are permuted so that none of the pattern's CAR, DOG, PUN or BYTE occurs?
2. **Define** Derangement. In how many ways can each of 10 people select a left glove and a right glove out of a total of 10 pairs of gloves so that no person selects a matching pair of gloves?
3. In **how many** ways one can arrange the letters of the word CORRESPONDENTS so that there are i) exactly 2 pairs of consecutive identical letters? ii) at least 3 pairs of consecutive identical letters? iii) no pair of consecutive identical letters?
4. Five teachers T_1, T_2, T_3, T_4, T_5 are to be made class teachers for five classes, C_1, C_2, C_3, C_4, C_5, one teacher for each class. $T_1$and T_2 do not wish to become the class teachers for C_1or C_2, T_3 and T_4 for C_4 or C_5, and T_5 for C_3 or C_4 or C_5. In **how many** ways can the teachers be assigned the work (without displeasing any teacher
5. **Find** the rook polynomial for the chess board as shown in the figure

6. (a) Solve the recurrence relation: $Cn = 3C_{n-1} - 2C_{n-2}, for\ n \geq 2$, given $C_1 = 5$, $C_2 = 3$
   (b) Solve the recurrence relation $a_{n+2} - 3a_{n+1} + 2a_n = 0$, $a_0 = 1$, $a_1 = 6$

MODULE 5
GROUP THEORY

1. Show that $(A, \cdot)$ is an abelian group where $A = \{a \in Q | a \neq -1\}$ and for any $a, b \in A$. $b = a + b + ab$.
2. State and prove Lagrange's theorem
3. If G be a group with subgroup H and K. If $|G| = 660$ and $|K| = 66$ and $K \subset H \subset G$ and find the possible value for $|H|$.
4. Show that i) the identity of G is unique. ii) the inverse of each element of G is Unique.
5. Define a group. Show that fourth roots of unity is an abelian group.
6. Define Klein 4 group .Verify A={1,3,5,7}is a Klein 4 group.
7. Let $G = S_4$, for $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, find the subgroup $H = <\alpha>$. Determine the left cosets of H in G
8. Define Cyclic group and show that $(G, 8)$ whose multiplicatio tableis as given below is Cyclic.

*	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	c	d	e	f	a
c	c	d	e	f	a	b
d	d	e	f	a	b	c
e	e	f	a	b	c	d
f	f	a	b	c	d	e